If the invalence of an integer is zero, Alanen calls it "untouchable". He conjectures that no odd number other than 5 is untouchable, and refers to this as a strengthened Goldbach conjecture (or "weakened", since if $n = p+q+1$, where p, q are distinct odd primes, then pq is a value of $s^{-1}(n)$, but there are in general other than "Goldbach" solutions to the equation $s(x) = n$). He gives the value of the invalence of n and a list of all solutions of $s(x) = n$ for $0 \leqq n \leqq 100$, the non-Goldbach solutions for $101 \leqq n \leqq 500$, and a list of the 570 untouchable numbers $2, 5, 52, 88, \cdots$ less than 5000.

Two sections develop algorithms for determining all untouchable numbers, and all cycles, below a given bound. Two others give the results of computer calculations (see next review) and a specification of the algorithms used. There are 33 references.

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1. RICHARD K. GUY & J. L. SELFRIDGE, "Interim report on aliquot series," *Proc. Winnipeg Conf. on Numerical Math.*, October 1971, pp. 557-580.

2. RICHARD K. GUY, D. H. LEHMER, J. L. SELFRIDGE & M. WUNDERLICH, "Second report on aliquot sequences," *Proc. Winnipeg Conf. on Numerical Math.*, October 1973.

3. H. J. J. TE RIELE, "A note on the Catalan-Dickson conjecture," *Math. Comp.*, v. 27, 1973, pp. 189-192.

4. Corrigendum, *ibid.*, p. 1011.

44 [9].—JACK ALANEN, *Tables of Aliquot Sequences*, 7 volumes, each of approximately 600 pages of computer output, filed in stiff covers and presented to the reviewer.

This is the output produced in connection with the author's thesis (see previous review). One volume investigates, by various algorithms, (certain subclasses of) partitions of n, and also carries out other algorithms designed to find all solutions of $s(x) = n$, where $s(x)$ is the sum of the divisors of x, other than x itself. A second volume continues the previous work, lists all n sequences with $n \leqq 48303$ for which $s^k(n) = 6$ for some k (the largest k in this range is 33) and lists the members of the aliquot sequence $s^k(138)$ for $0 \leqq k \leqq 112$ (it was earlier shown by D. H. Lehmer that $s^{177}(138) = 1$, the maximum term being $s^{117}(138) = 179931895322$). The other five volumes give all terms of all n sequences for $1 \leqq n \leqq 10000$, $10001 \leqq n \leqq 20000$, $20001 \leqq n \leqq 30000$, $30001 \leqq n \leqq 40000$, $40001 \leqq n \leqq 48303$ and the rank of the bounding term, where the bounding term is either 1, or a member of a cycle, or the first term of the sequence which exceeds 10^{10}. Of the sequences associated with the first 40,000 integers, 33450 terminate at 1, 5676 exceed Alanen's bound of 10^{10}, 325 become periodic at a perfect number (6, 496 or

8128), 495 become periodic with period 2 (amicable pair), and 54 lead into one of the two Poulet cycles.

For a review of Paxson's related tables see *Math. Comp.*, v. 26, 1972, UMT **38**, pp. 807-809.

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45 [**10**].—P. A. MORRIS, *Characteristic Polynomials of Trees on up to* 14 *Nodes*, University of the West Indies, St. Augustine, Trinidad, West Indies, December 1973. Ms of 8 pp. + 57 computer sheets deposited in the UMT file.

Herein are listed the coefficients of the characteristic polynomials of the adjacency matrices of all trees with 13 or fewer nodes.

The list was generated in two ways to provide a check on the calculations, which were performed on a 1 CL 1902A and an IBM 1620, respectively. The first method employs a theorem of Collatz and Singowitz [1], which asserts that if $\phi(T) = \sum_{k=0}^{n} (-1)^k a_k \lambda^{n-k}$ is the characteristic polynomial of a tree T on n nodes, then $a_{2k+1} = 0$ and a_{2k} equals the number of ways of finding k mutually nonadjacent edges in T. The second method uses a known decomposition theorem [2], which states that if u is a node of valency 1 connected to a node v, $T - uv$ is the tree (together with the isolated node u) formed by deleting the edge uv, and $T - u - v$ is the forest formed by deleting nodes u and v and their incident edges, then $\phi(T) = \phi(T - uv) - \phi(T - u - v)$.

A further check of the accuracy of the list was made by comparison with the corresponding data in the table of Mowshowitz [3], which includes all trees on 10 or fewer nodes.

AUTHOR'S SUMMARY

1. L. COLLATZ & U. SINGOWITZ, "Spektren endlichen Graphen," *Abh. Math. Sem. Univ. Hamburg*, v. 21, 1957, pp. 63-77.

2. F. HARARY, C. KING, A. MOWSHOWITZ & R. C. READ, "Cospectral graphs and digraphs," *Bull. London Math. Soc.*, v. 3, 1971, pp. 321-328.

3. A. MOWSHOWITZ, "The characteristic polynomial of a graph," *J. Combinatorial Theory Ser. B*, v. 12, 1972, pp. 177-193.

46 [**12**].—LOUIS D. GREY, *A Course in* APL/360 *with Applications*, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1973, xviii + 332 pp., 24 cm. Price $7.50 (paperbound).

If this paper-back book were used merely as a reference manual for APL programmers, it would serve a useful function since it is well organized, comprehensive and well documented. But the text is far more than a work of reference. It is an excellent vehicle for teaching this most elegant and succinct language, one which is considered by some to be a serious competitor with